CHEMICAL ENGINEERING TRANSACTIONS

VOL. 112, 2024

A publication of AIDIC

The Italian Association of Chemical Engineering
Online at www.cetjournal.it

Guest Editors: Selena Sironi, Laura Capelli

ISBN 979-12-81206-13-7; **ISSN** 2283-9216

DOI: 10.3303/CET24112050

Natech Risk Assessment with Imprecise Probabilities

Alireza Dehghanisanij, Nima Khakzad *

School of Occupational and Public Health, Toronto Metropolitan University, Toronto, Canada
nima.khakzad@torontomu.ca

Natechs are technological accidents that are triggered by natural disasters. The increase in the frequency and severity of climatic natural disasters and the growth of industrialization demand the development of dedicated methodologies for risk assessment and management of Natechs. Due to the lack of accurate and sufficient data, the risk assessment of Natechs has largely been based on subjective assumptions and imprecise probabilities, making the assessed risks and the subsequent risk management strategies less cost-effective. In the present study, evidence theory, as an effective technique for dealing with imprecise probabilities, and Bayesian network, as an effective tool for reasoning under uncertainty, are combined to develop a methodology for risk analysis of Natechs. Flotation of oil tanks during floods has been considered to exemplify the methodology. It is demonstrated that replacing the interval probabilities with average probabilities can result in different yet consistent risk values.

1. Introduction

Technological accidents which are triggered by natural disasters are known as natural-technological accidents or Natechs. Natechs that occur to chemical and process plants can be catastrophic due to the possibility of damage to process units and subsequent release of hazardous chemicals which may cause fire and explosions or major environmental pollution. Among the process units, atmospheric storage tanks have reportedly been the most vulnerable type of vessels (Godoy, 2007). This is because such vessels have thin shells and high volume-weight ratios. A thin shell makes the storage tank very susceptible to lateral forces exerted by high winds or floods, whereas a high volume-weight ratio makes the tank susceptible to buoyancy force in the event of floods or heavy rainfalls (Godoy, 2007; Qin et al., 2020). Flotation of atmospheric storage tanks due to the buoyancy force has been identified as the most common failure mode during floods (Cozzani et al., 2010; Landucci et al., 2012; Khakzad and van Gelder, 2017, 2018).

Compared to conventional technological accidents which are caused by random failures or human error, risk assessment and management of natechs are prone to more uncertainty and are thus more challenging. The foregoing uncertainty consists of aleatory uncertainty that arises from the randomness of natural disasters or failures and epistemic uncertainty that represents our lack of knowledge due to insufficient or inaccurate objective data for the Natech of interest.

Probability theory has effectively been used to account for uncertainty embedded in the occurrence and severity of natural disasters as well as the extent of damage they may cause to structures and industrial plants. However, in the absence of sufficiently large and reliable datasets or accurate field measurements, subject matter experts may inevitably come up with subjective imprecise probabilities that may influence the accuracy and credibility of risk analysis if not properly handled. Evidence theory (Dempster, 1967; Shafer, 1976) is an effective tool for handling imprecise probabilities. In Evidence theory, the propagation of uncertainty is based on belief masses rather than probability masses. Belief masses are analyst's degrees of belief about a hypothesis and can be derived from imprecise probabilities. Compared with probability theory, the application of evidence theory to the domain of risk assessment and management has not been so widespread mainly due to a lack of efficient inference algorithms. Simon and Weber (2009) and Khakzad (2019) demonstrated that the Bayesian network (BN) and evidence theory can be combined to form a so-called evidential network (EN), where BN is used to handle belief masses developed through evidence theory the same way as it handles the probabilities.

Paper Received: 11 June 2024; Revised: 08 September 2024; Accepted: 02 October 2024
Please cite this article as: Dehghanisanij A., Khakzad N., 2024, Natech Risk Assessment with Imprecise Probabilities, Chemical Engineering Transactions, 112, 295-300 DOI:10.3303/CET24112050

The present study aims to demonstrate an application of EN to risk assessment and management of natechs when due to a lack of knowledge the analyst may express his uncertainty in the form of interval probabilities. Section 2 briefly reviews the evidence theory and how it can be combined with BN; in Section 3 the methodology is applied to risk assessment of tank flotation during floods; Section 4 concludes the work.

2. Evidential theory and evidential network

In this section, the basics of evidence theory and evidential networks are briefly reviewed.

2.1 Evidence theory

Using the evidence theory (Dempster, 1967; Shafer, 1976), all the possible states of a random variable can be presented in a set, named the frame of discernment Ω. To each subset of Ω such as A_i, which is a hypothesis about the state of the variable, a weight $0.0 \leq m(A_i) \leq 1.0$ can be assigned to express the degree of belief, based on objective data or subjective opinion, in the claim that the variable' state belongs to A_i (Rakowsky, 2007). Having $m(A_i)$, which is also known as the belief mass of A_i, the belief $bel(A_i)$ and plausibility $pls(A_i)$ can be determined. For the sake of clarity, consider a binary component X with the two states, fail and work, and thus a frame of discernment as Ω_X = {fail, work}. Therefore, the set of all the subsets of Ω_X would be A: {{∅}, {fail}, {work}, {fail, work}}, where A_1 = {∅}, A_2 = {fail}, A_3 = {work}, and A_4 = {fail, work}. Each member of A for which $m(A_i) > 0$ is called a focal set. If we are certain that all the states of the variable are included in the frame of discernment, then m(∅) = 0. It must always hold that:

$$\sum_{A_i} m(A_i) = 1 \tag{1}$$

Having the belief masses determined, the belief and plausibility measures of each focal set can be defined using:

$$bel(A_i) = \sum_{A_j|A_j \subseteq A_i} m(A_j) \tag{2}$$

$$pls(A_i) = \sum_{A_j|A_j \cap A_i \neq \emptyset} m(A_j) \tag{3}$$

For instance, an expert may assign the weights m_X({fail}, {work}, {fail, work}) = (0.15, 0.8, 0.05), in which m_X({fail, work}) = 0.05 refers to the expert's uncertainty about the state of X. Using Equations (2) and (3), the belief and plausibility of X = {fail} can be calculated, respectively, as bel(X = {fail}) = m({fail}) = 0.15 and pls(X = {fail}) = m({fail}) + m({fail, work}) = 0.15 + 0.05 = 0.2. It should be noted that in calculating the plausibility of X = {fail} using Equation (3), the mass of {fail, work} should be considered because {fail} ∩ {fail, work} ≠ ∅; however, it should not be considered in calculating the belief of X = {fail} via Equation (2) because {fail, work} ⊄ {fail}.
Further, the amount of uncertainty $Unc(A_i)$ of a focal set can be expressed as the difference between $pls(A_i)$ and $bel(A_i)$ as (Rakowsky, 2007):

$$Unc(A_i) = pls(A_i) - bel(A_i) \tag{4}$$

Since m_X({fail, work}) represents the uncertainty about the state of X, Equation (4) can be used to calculate it as m_X({fail, work}) = pls(X = {fail}) – bel(X = {fail}) = 0.05. Subsequently, $bel(A_i)$ and $pls(A_i)$, which are non-additive, can be taken as lower and upper probability bounds of A_i, respectively (Shafer, 1976):

$$bel(A_i) \leq P(A_i) \leq pls(A_i) \tag{5}$$

$$bel(A_i^c) = 1 - pls(A_i) \tag{6}$$

$$pls(A_i^c) = 1 - bel(A_i) \tag{7}$$

where A_i^C is the complement of Ai in the sense that $A_i^c = \Omega - A_i$.
According to Equation (5), 0.15 ≤ P(X = fail) ≤ 0.2. Moreover, according to Equations (6) and (7), bel(X = {work}) = 1 – pls(X = {fail}) = 0.8 and pls(X = {work}) = 1 – bel(X = {fail}) = 0.85, and thus 0.8 ≤ P(X = work) ≤ 0.85. Having the bel and pls functions, the belief mass of a focal set can be determined using the möbius transformation as (Smets, 2002):

$$m(A_i) = \sum_{A_j|A_j \subseteq A_i} (-1)^{|A_i - A_j|} \, bel(A_j) \tag{8}$$

where $|A_i - A_j|$ refers to the difference between the number of elements of A_i and A_j.

2.2 Evidential network

Simon and Weber (2009) and Khakzad (2019) showed that belief masses can be used in BN the same way as the probabilities and thus the algorithms developed for BN could be employed to propagate belief masses in a system. Since the belief masses allocated to the focal sets of each random variable must add up to 1.0, they can be treated as marginal probabilities for the root nodes in a BN.
Considering a parallel system in Figure 1, comprising two binary components X and Y. Further, assume that due to a lack of sufficient knowledge, the analyst cannot assign precise probabilities to the states of X and Y and decides to express their uncertainty in the form of interval probabilities as 0.15 ≤ P(X = fail) ≤ 0.35 and 0.2 ≤ P(Y = fail) ≤ 0.5.

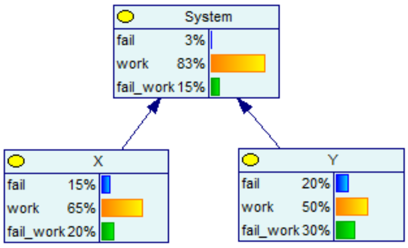

Figure 1. BN for failure assessment of System using belief masses of X and Y. Nodes X and Y are connected to node System by AND agte.

Having these interval probabilities, the belief masses of the focal sets of X and Y can be identified. For instance, consider P(X = fail), where its lower and upper bounds can be taken as the *bel* and *pls* functions, respectively:

- bel(X = {fail}) = 0.15 → m_X({fail}) = 0.15
- bel(X = {work}) = 1 – pls(X = {fail}) = 1 – 0.35 = 0.65 → m_X({work}) = 0.65
- m_X({fail, work}) = 1 – m_X({fail}) – m_X({work}) = 1 – 0.15 – 0.65 = 0.2
- As a result: m_X({fail}, {work}, {fail, work}) = (0.15, 0.65, 0.2).

Following the same procedure, m_Y({fail}, {work}, {fail, work}) = (0.2, 0.5, 0.3). The calculated mass beliefs can now be used in BN to compute the belief masses of the system. In the BN shown in Figure 1, the focal sets of X and Y have been considered as the states of nodes X and Y while the respective belief masses have been considered as their probabilities. The same focal sets have also been considered for the node System, which is connected to nodes X and Y via AND gate. The truth table shown in Table 1 can be used to populate the conditional belief table of node System (Simon and Weber, 2009)

Table 1. Truth table used to populate the conditional belief table of node System in Figure 1 in case of AND gate and OR gate.

Component		System: AND gate			System: OR gate		
X	Y	{fail}	{work}	{fail, work}	{fail}	{work}	{fail, work}
{fail}	{fail}	1	0	0	1	0	0
{fail}	{work}	0	1	0	1	0	0
{fail}	{fail, work}	0	0	1	1	0	0
{work}	{fail}	0	1	0	1	0	0
{work}	{work}	0	1	0	0	1	0
{work}	{fail, work}	0	1	0	0	0	1
{fail, work}	{fail}	0	0	1	1	0	0
{fail, work}	{work}	0	1	0	0	0	1
{fail, work}	{fail, work}	0	0	1	0	0	1

The developed BN can accordingly be used to calculate the belief masses of node System. Having the belief masses of System as m_{System}({fail}, {work}, {fail, work}) = (0.03, 0.825, 0.145), the bel and pls functions or the lower and upper bound probabilities of (System = fail) can be calculated as bel(System = {fail}) = 0.03 and pls(System = {fail}) = 0.03 + 0.145 = 0.175, resulting in 0.03 ≤ P(System = fail) ≤ 0.175.

3. Risk assessment with imprecise probabilities

In this section, using an illustrative case study, the development of the methodology is demonstrated, and the results are presented and discussed regarding the previous studies.

3.1 Case study

Flotation of oil storage tanks has reportedly been the most frequent failure mode during floods. Flotation of storage tanks occurs if the upthrust force of flood (buoyancy force F_B) exceeds the bulk weight of the storage tank (weight of the tank W_T plus the weight of its containment W_L). Considering that such tanks are usually unanchored and thus no resisting force is exerted on them from their foundation, F_B, W_T and W_L are the only forces considered for the flotation of the tank in Figure 2 (Khakzad and van Gelder, 2017; Dehghanisanij et al., 2024). Given the tank's dimension and the flood's inundation depth, the foregoing forces can be modeled as:

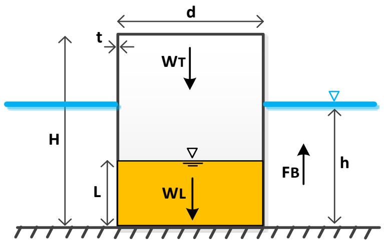

Figure 2. Schematic of flotation-related loading and resisting forces acting on an oil storage tank.

$$F_B = \rho_w\, g \frac{\pi\, d^2}{4} h \tag{9}$$

$$W_T = \rho_s\, g \left(\pi d H + 2\frac{\pi\, d^2}{4}\right) t \tag{10}$$

$$W_L = \rho_l\, g \frac{\pi\, d^2}{4} L \tag{11}$$

Consider a case in which the characteristics of the storage tank and an imminent flood are measured or predicted as listed in Table 2.

Table 2. Parameters used to develop the limit state equation for flotation of the storage tank.

Parameters	Symbols	Values
Tank's height	H (m)	6
Tank's diameter	d (m)	10
Tank's shell thickness	t (m)	0.01
Chemical inventory height	L (m)	(0.5, 1.0, 1.5)
Tank material density (steel)	ρ_s (kg/m^3)	7900
Flood water density	ρ_w (kg/m^3)	1024
Chemical inventory density (gasoline)	ρ_l (kg/m^3)	850
Flood inundation height	h (m)	N (μ = 1, σ = 0.2)

Given the foregoing forces, the limit-state equation (LSE) for the flotation of the tank can be developed (Landucci et al., 2012; Khakzad and van Gelder, 2017):

$$\mathrm{LSE} = F_B - W_T - W_L \tag{12}$$

As such, the flotation probability of the tank can be presented as the probability that LSE > 0.
Based on historical data, the flood's inundation height is expected to follow a normal distribution (Table 2). Further, assume that although the specific values of the tank's dimension are known, the initial amount of crude oil inside the tank is unknown as the mechanical and automatic level indicators each show a different number: the mechanical level indicator, which is likely to malfunction and is thus not so reliable, shows the crude level as L = 1.0 m but the automatic gauge shows the level as L = 0.5 m. Therefore, the operator decides to take a glance at the crude level via the top manhole, estimating the crude level as L = 1.5 m. As such, the operator's uncertainty about the level of crude oil can be modeled as a tertiary random variable L with three states as L_1 = 0.5 m, L_2 = 1 m, and L_3 = 1.5 m, with the following interval probabilities based on his confidence in the mechanical and automatic gauges and his own estimate:

- $0.2 \le P(L_1) \le 0.5$

- $0.3 \le P(L_2) \le 0.5$
- $0.2 \le P(L_3) \le 0.3$.

3.2 Risk assessment

According to the parameters in Table 2, the magnitudes of the three forces can be calculated as $W_T = 268$ (KN), $W_L = 655 \times L$ (KN), and $F_B = 789 \times h$ (KN). The probability that the storage tank floats due to the buoyancy force can thus be calculated as:

$$\mathrm{P(Flotation = yes) = P(F_B > W_T + W_L) = P(789\,h > 268 + 655\,L) = P}\left(\mathrm{h} > \frac{268 + 655\,\mathrm{L}}{789}\right) \tag{13}$$

Considering L as an uncertain variable with three states as L_1, L_2 and L_3, its frame of discernment can be developed as $\Omega_L = \{L_1, L_2, L_3\}$. Consequently, the set of its focal sets would be A_L: $\{\{L_1\}, \{L_2\}, \{L_3\}, \{L_1, L_2\}, \{L_1, L_3\}, \{L_2, L_3\}, \{L_1, L_2, L_3\}\}$. The belief mass of each focal set can subsequently be determined. For example, for the first focal set $\{L_1\}$ with the lower and upper bound probabilities as $0.2 < P(L_1) < 0.5$, the belief and plausibility functions can be determined as $bel(\{L_1\}) = 0.2$ and $pls(\{L_1\}) = 0.5$. Since $\{L_1\}$ is a singleton, $m(\{L_1\}) = bel(\{L_1\}) = 0.2$. Similarly, $m(\{L_2\}) = 0.3$ and $m(\{L_3\}) = 0.2$.
Further, consider the focal set $\{L_1, L_2\}$. Since $\{L_1\}$, $\{L_2\}$, and $\{L_1, L_2\}$ are all the subsets of $\{L_1, L_2\}$, we will have $m(\{L_1, L_2\}) = bel(\{L_1, L_2\}) - bel(\{L_1\}) - bel(\{L_2\})$. Furthermore, $bel(\{L_1, L_2\}) = 1 - pls(\{L_3\}) = 1 - 0.3 = 0.7$. As a result, $m(\{L_1, L_2\}) = 0.7 - 0.2 - 0.3 = 0.2$. Following the same procedure, $m\{\{L_1\}, \{L_2\}, \{L_3\}, \{L_1, L_2\}, \{L_1, L_3\}, \{L_2, L_3\}, \{L_1, L_2, L_3\}\} = (0.2, 0.3, 0.2, 0.2, 0.1, 0.0, 0.0)$. Since $m(\{L_1, L_3\}) = m(\{L_1, L_2, L_3\}) = 0.0$, they would not be considered focal sets anymore.
As can be seen from Equation (13), the only influential parameters in estimating the probability of tank flotation are the flood inundation height (h) and the height of the chemical inside the tank (L). To facilitate the propagation of uncertainty – aleatory uncertainty in h and epistemic uncertainty in L – the BN in Figure 3 can be developed.

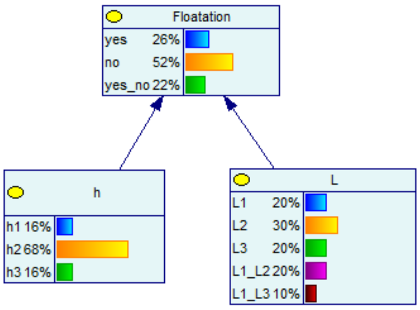

Figure 3. BN to estimate the probability of tank floatation.

Quantifying the BN, the belief masses of the focal sets of "Flotation" can be calculated as $m_{Flotation}$ ({yes}) = 0.26, $m_{Flotation}$ ({no}) = 0.52, and $m_{Flotation}$ ({yes, no}) = 0.22. Consequently, the belief and plausibility functions or the lower and upper bound probabilities of (Flotation = yes) can be calculated as: $0.26 \le P(Flotation = yes) \le 0.48$. Likewise, $0.52 \le P(Flotation = no) \le 0.74$.

3.3 Discussion

Regarding the probability intervals for L, each probability interval may be replaced with an average probability. Yager and Kreinovich (1999) proposed that the average probability $p_j^{\sim}$ for an interval $p_j = [p_j^-, p_j^+]$ can be calculated as:

$$p_j^{\sim} = \frac{\Sigma^+ - 1}{\Sigma^+ - \Sigma^-} \cdot p_j^- + \frac{1 - \Sigma^-}{\Sigma^+ - \Sigma^-} \cdot p_j^+ \tag{14}$$

where:

$$\Sigma^+ = \sum_{i=1}^{n} p_i^+ \tag{15}$$

$$\Sigma^- = \sum_{i=1}^{n} p_i^- \tag{16}$$

p_j^- and p_j^+ are the lower and upper probability bounds of the interval, respectively. Therefore, given the probability intervals for L as $0.2 \le P(L_1) \le 0.5$, $0.3 \le P(L_2) \le 0.5$, and $0.2 \le P(L_3) \le 0.3$, the values of Σ^+ and Σ^- can be calculated as:

- $\Sigma^{+} = \sum_{i=1}^{3} p_i^{+} = p_1^{+} + p_2^{+} + p_3^{+} = 0.5 + 0.5 + 0.3 = 1.3$
- $\Sigma^{-} = \sum_{i=1}^{3} p_i^{-} = p_1^{-} + p_2^{-} + p_3^{-} = 0.2 + 0.3 + 0.2 = 0.7$

Subsequently, the average probabilities for L_1, L_2, and L_3 can be calculated as $p_1^{\sim} = 0.35$, $p_2^{\sim} = 0.4$, and $p_3^{\sim} = 0.25$. Now, *L* can be considered as a discrete variable with a discrete distribution as P(L_1, L_2, L_3) = (0.35, 0.4, 0.25). Having *h* as a normal variable (Table 2) with a normal distribution as h ~ N(1, 0.2), Monte Carlo simulation can be employed to estimate the probability of flotation via Equation (13). Conducting the simulation for 10,000 iterations, the probability of flotation can be estimated as P(Flotation = yes) = 0.38, and subsequently P(Flotation = no) = 1 – 0.38 = 0.62. The results derived from evidence theory and the ones from the average probabilities are summarized in Table 3.

Table 3. Comparison between the results obtained from interval probabilities and average probabilities.

Approach	P(Flotation = yes)	P(Flotation = no)
Interval probabilities	[0.26, 0.48]	[0.52, 0.74]
Average probabilities	0.38	0.62

4. Conclusions

In the present study, we presented an application of evidence theory to risk assessment of oil storage tanks during floods. It was demonstrated that replacing the interval probabilities with average probabilities may result in different yet consistent risk outcomes. Therefore, in the absence of required resources (time, expertise, etc.) for dealing with interval probabilities, the analyst may decide to replace them with average probabilities as the results are still consistent with the results obtained from the interval probabilities. However, more applications and comparisons are required to determine if the average probabilities could efficiently substitute the interval probabilities for risk assessment and if both approached would lead to similar risk management strategies.

References

Cozzani V., Campedel M., Renni E., Krausmann E., 2010, Industrial accidents triggered by flood events: analysis of past accidents, Journal of Hazardous Materials, 175, 501-509.

Dehghanisanij A., Khakzad N., Salzano E., Amyotte P., 2024, Protecting oil storage tanks against floods: Natech risk assessment with imprecise probabilities, The Canadian Journal of Chemical Engineering, 102(10), 3333-3344.

Dempster A.P., 1967, Upper and lower probabilities induced by a multivalued mapping, The Annals of Mathematical Statistics, 38, 325-339.

Godoy L.A., 2007, Performance of storage tanks in oil facilities damaged by Hurricanes Katrina and Rita, Journal of Performance of Constructed Facilities, 21(6), 441-449.

Khakzad, N., 2019, System safety assessment under epistemic uncertainty: using imprecise probabilities in Bayesian network, *Safety science*, *116*, 149-160.

Khakzad N., van Gelder P., 2017, Fragility assessment of chemical storage tanks subject to floods, Process Safety and Environmental Protection, 111, 75-84.

Khakzad, N. and Van Gelder, P., 2018, Vulnerability of industrial plants to flood-induced natechs: A Bayesian network approach, Reliability Engineering & System Safety, 169, pp.403-411.

Landucci G., Antonioni G., Tugnoli A., Cozzani V., 2012, Release of hazardous substances in flood events: damage model for atmospheric storage tanks, Reliability Engineering and System Safety, 106, 200-216.

Qin, R., Khakzad, N. and Zhu, J., 2020, An overview of the impact of Hurricane Harvey on chemical and process facilities in Texas, International journal of disaster risk reduction, 45, p.101453.

Rakowsky U.K., 2007, Fundamentals of the Dempster-Shafer theory and its applications to system safety and reliability modelling, in Proc. of the ESRA Summer Safety and Reliability Seminar – SSARS 2007, Sopot, Poland, July 2007.

Shafer G., 1976, A Mathematical Theory of Evidence. Princeton: Princeton University Press.

Simon C., Weber P., 2009, Evidential networks for reliability analysis and performance evaluation of systems with imprecise knowledge, IEEE Transactions on Reliability, 58(1), 69-87.

Smets P., 2002, The application of matrix calculus to belief functions, International Journal of Approximate Reasoning, 31, 1-30.

Yager RR., Kreinovich V., 1999, Decision making under interval probabilities, International Journal of Approximate Reasoning, 22(3), 195-215.