

# A Bilevel Integer Programming Approach for a Stackelberg Game on Sustainable Aviation Fuel Production Optimization

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Along with the encouragement of government sectors to utilize sustainable aviation fuel, production firms must ensure that environmental sustainability is gained together with economic sustainability. Sustainable aviation fuel has been proven effective by multiple studies to alleviate the carbon emissions that the aviation industry brings to the environment. The biggest hindrance of its full adoption is its high cost. Governments need to develop strategies which will encourage industries to adopt the use of sustainable aviation fuel. This government-industry interaction when considered, can facilitate the development of optimal environmental policies from government which can influence industry to align its decisions towards the government's goals. This work uses a bilevel integer programming approach to develop a Stackelberg or leader-follower game model for the production of sustainable aviation fuel. The model considers different production pathways while taking into account environmental (government's goal) and economic (industry's goal) sustainability. To implement and test the optimization model, an illustrative case study is used. Results show that incentives are useful in encouraging the follower to compromise and choose the Alcohol to Jet (ATJ) pathway even though they do not fully benefit to the decision. The optimal solution mix from the illustrative case study resulted in a reduction of 20,109 M t CO<sub>2</sub> reduction and an increase in cost of 906,014 M EUR over a period of ten years.

## 1. Introduction

With the ongoing global growth in the demand for air travel, the aviation industry has been one of the fastest-growing contributors of greenhouse gas emissions (Sher et al., 2021). There is now an increasing pressure to adopt net-zero emission practices in the aviation industry. One such strategy is the use of sustainable aviation fuel (SAF). It facilitates the gradual transition of many airlines from using conventional aviation fuel (CAF) to SAF. SAF contributes significantly to the reduction of greenhouse gas emissions as it is based on renewable resources compared to CAF as it is produced from fossil-based resources. Several pathways have been explored to produce SAF as it can be obtained from different types of feedstocks. These options all have their own advantages and disadvantages. Deciding on which technology pathway to invest in is important to ensure that maximum benefit obtained given resource limitations.

Decision support systems have been developed to aid the decision-making process to produce SAF, these include techniques for determining the best-performing feedstocks (Markatos and Pantelakis, 2022) and identifying the optimal process of implementing and producing SAF for a fuel production firm (Ahmad et al., 2021). A multi-criteria decision analysis was done by Okolie et al. (2023) to choose a SAF production pathway to invest on while considering economic, environmental, and technological criteria. Similarly, Wang et al. (2021) simulated the economic benefits of different policies which could be implemented by the government in the production of SAF; despite the significant findings on all of the proposed policies that allowed production firms to break even, no studies have considered the strategic interaction between the government and industry in optimizing the production of SAF. By allowing the government to intervene in the supply chain of aviation fuel through the provision of monetary incentives in the production of SAF, this could lead to SAF being a competitive

alternative to CAF (Sharma et al., 2021). In addition, Sharma et al. (2021) also discussed how implementing adequate incentives for the production and usage of SAF reduced carbon emissions from airplanes by 65 %. In a similar study on transitioning to alternative fuel vehicles, Hachem (2023) suggests that adequate government intervention will lead to increased environmental awareness for consumers and had little effect on the profits of manufacturers. The absence of government intervention conversely hinders the transition to more environmentally friendly alternatives and increases carbon emissions in the long term.

The nature of this study leads to the formulation of a decision support system that selects the best alternative amongst different alternatives given a set criteria (Tapia, 2021). The decision support system would quantify the effects of each decision to select the optimal decision based on the criteria. Optimization models that involve government-industry interactions can be formulated using a Stackelberg or leader-follower game (Tan et al., 2021). A Stackelberg game depicts the economics-inspired sequential decision-making process between the government and the industry (Von Stackelberg, 2011), which has yet to be considered in the literature and can best illustrate the case for the production of SAF. In this non-symmetric game, a leader would make a decision first before the follower responds to the decision (Yan et al., 2022). This approach is considered in this study. Failure to consider this approach may lead to sub-optimal solutions wherein the government's or the industry's goals are not considered. The Stackelberg game can be modelled through bilevel programming, wherein the lower-level objective function (follower's objective) is nested within the upper-level (leader's objective function) optimization model as a constraint (Bard, 1998). The optimization model aims to aid the process of selecting the optimal pathway for producing SAF, which can be modelled using 0-1 programming or binary decision variables (Kantardgi et al., 2006).

## 2. Problem Statement

This study aims to determine the Stackelberg strategy that minimizes environmental impact to the cost-minimizing strategy of investing in a SAF production setup from the current CAF production setup. Given:

- A set of SAF production pathway options that the current CAF plant can be converted into, each with different techno-economic parameters.
- The government (leader) aims to maximize the total reduction in carbon emissions from the old CAF setup to the new SAF setup. Emissions include those generated from setting-up the process (fixed emissions) and those generated from production which is proportional to the production capacity (variable emissions).
- The industry (follower) aims to rationally minimize the total cost increase from the old CAF setup to the new SAF setup. Costs include setup costs, operating costs, and feedstock costs based on a specific amount of SAF required to be produced. The overall cost is reduced by available government incentives when industry selects the same technology as the government.

## 3. Model Formulation

The model is formulated considering that the input parameters are assumed to be deterministic in nature. The solution for the bilevel optimization problem, specifically with the Stackelberg game, assumes a self-optimal response of the industry following the leader's optimal response (Tapia et al., 2023).

The objective function of the leader, in this case the government institution, is defined by Eq(1) and aims to maximize the total carbon emission reduction based on the difference in the variable emissions of the chosen pathway from the old CAF setting. The setup emissions of the new pathway is also considered. The objective function of the follower, in this case the SAF production firm, is nested within the leader's objective function and is considered to be a constraint of the bilevel optimization model (Eq(2)). It seeks to minimize the total cost increase from the old CAF production setup to the new SAF production setup of the follower. Cost increase include the operating costs and feedstock costs from the old CAF setting to the new SAF setting. The setup cost of the new SAF setting and the incentives given if the government chooses to incentivise the pathway that the firm has chosen are also considered. Eq(3) and Eq(4) assures that both the leader and the follower would choose only one pathway among the available options. Eq(5) and Eq(6) formally defines the binary nature of the decision variables.

$$L = \text{Max CO}_2 \text{ Reduction} = \text{OVE} \cdot \text{SAF} - \sum_i \text{NVE}_i \cdot \text{SAF} \cdot x_i \cdot y_i - \sum_i \text{SE}_i \cdot x_i \cdot y_i \quad (1)$$

subject to:

$$F = \text{Min Cost Increase} = (\sum_i \text{NOC}_i \cdot \text{SAF} \cdot y_i - \text{OOC} \cdot \text{SAF}) + (\sum_i \text{NFC}_i \cdot \text{SAF} \cdot y_i - \text{OFC} \cdot \text{SAF}) \\ + (\sum_i \text{SC}_i \cdot y_i) - (\sum_i \text{IC}_i \cdot x_i \cdot y_i) \quad (2)$$

$$\sum_i x_i = 1 \quad \forall i \quad (3)$$

$$\sum_i y_i = 1 \quad \forall i \quad (4)$$

$$x_i \in \{0,1\} \quad \forall i \quad (5)$$

$$y_i \in \{0,1\} \quad \forall i \quad (6)$$

The nonlinear nature of the model can be seen with the multiplication of two decision variables  $x_i$  and  $y_i$ , making it a bilinear product. To ensure that the solution that will be obtained is the global optima, the nonlinearity of the model is addressed by defining the quadratic terms into single terms, defined by Eq(7). Eq(8) and Eq(9) represent the activating constraint for the replaced quadratic term  $z_i$ , ensuring that it is only activated when both  $x_i$  and  $y_i$  are activated. To formally introduce the binary nature of the dummy variable  $z_i$ , Eq(10) is introduced.

$$x_i \cdot y_i = z_i \quad \forall i \quad (7)$$

$$x_i + y_i \leq 1 + z_i \quad \forall i \quad (8)$$

$$x_i + y_i \geq 2z_i \quad \forall i \quad (9)$$

$$z_i \in \{0,1\} \quad \forall i \quad (10)$$

#### 4. Solution Approach

In a leader-follower game, the aim is to find a compromise between two conflicting objective functions of the leader and the follower. In other words, the goal is to minimize compromising the trade-offs of the leader and the follower from a solution that suits both and a solution that does not consider the other's benefit (Uy et al., 2023). The Approximate Stackelberg Strategy is used in solving the nested optimization model. The strategy uses an interactive fuzzy optimization algorithm that was first developed by Emam (2006) and was further modified by Tan and Aviso (2022) to handle binary variables. Tan et al. (2023) used the method to solve a similar bilevel optimization model and has been proven it to be effective. The method ensures that a balance is attained in the leader's and the follower's objective, attempting to maximize the worse possible value until both have reached a solution that compromises the least (Solis and San Juan, 2021), as seen in Eq(11). This objective function is used, replacing the objective functions of the leader and the follower. To linearize Eq(11), the new objective function that replaces it is presented in Eq(12), with the linearization constraints shown in Eq(13) and Eq(14).

$$\text{Max } Z = \min\left\{\frac{L-L^*}{L^{**}-L^*}, \frac{F^*-F}{F^*-F^{**}}\right\} \quad (11)$$

$$\text{Max } Z = \lambda \quad (12)$$

$$\lambda \leq \frac{L-L^*}{L^{**}-L^*} \quad (13)$$

$$\lambda \leq \frac{F^*-F}{F^*-F^{**}} \quad (14)$$

#### 5. Illustrative Case Study

The case study is based on a production firm of aviation fuel that plans to decide on the optimal production technology for SAF. The major investment decision involves several factors, such as the capital expenditures, operating expenditures, setup emissions, and operating emissions, derived from the study of Ng et al. (2021). Feedstock costs are also included in the parameters, derived from the study of Shahriar and Khanal (2022). Different pathways have different advantages and disadvantages to each factor. Moreover, the government can support one of the SAF pathways by incentivizing industries to adopt of the technology. For this case study, five pathways are given and intermediate calculations were done, Alcohol to Jet (ATJ) (Tanzil et al., 2021), Synthesized iso-paraffinic (SIP) (Qiu et al., 2019), Fischer-Tropsch (FT) (Doustdar et al., 2016), Hydrotreated Esters and Fatty Acids (HEFA) (Zanata et al., 2019), and fast pyrolysis (FP) (Carrasco et al., 2017).

Table 1: Techno-economic parameters of SAF Pathway Options

Categories	ATJ	SIP	FT	HEFA	FP
Capital Expense (EUR)	270,314,400	251,160,000	629,740,000	147,476,000	268,180,000
Operating Expense (EUR/m <sup>3</sup> )	9.54	2.83	0.56	0.61	0.68
Feedstock Cost (EUR/m <sup>3</sup> )	252.76	183.42	161.06	478.34	162.79
Incentive (EUR)	138,000	92,000	115,000	115,000	92,000
Variable Emissions (kg CO <sub>2</sub> /m <sup>3</sup> )	263.72	2,851.08	784.05	1,201.02	295.80
Setup Emissions (kg CO <sub>2</sub> )	393,757.92	649,392.92	384,910.39	223,832.40	548,392.83

For this illustrative case study, it is assumed that the different pathways are not mutually exclusive, and no interaction is present among the different pathways. The total SAF to be produced in the relevant period of study is set at 3,918,394,411 m<sup>3</sup> for ten years. The operating cost for the current CAF pathway is EUR 0.36 per m<sup>3</sup> (Goldstein, 2021). The feedstock cost for the current CAF pathway is EUR 30.96 per m<sup>3</sup> (Goldstein, 2021). The current variable emissions that the CAF pathway produces is 5,395.66 kg CO<sub>2</sub>/m<sup>3</sup> (International Aviation Transport Authority, 2023). The incentives were identified based on the carbon emissions each pathway produces. The other data used for this illustrative case study are presented in Table 1. More comprehensive data for the illustrative case study can be provided upon request.

Table 2: Player's Preferred Solutions and Approximate Stackelberg Strategy

	Leader's Preferred Solution	Follower's Preferred Solution	Approximate Stackelberg Strategy
Emission Reduction (t CO <sub>2</sub> )	20,108,954,960	18,070,130,329	20,108,954,960
Variable Emission Reduction (t CO <sub>2</sub> )	20,108,955,354	18,070,130,714	20,108,955,354
Setup Emissions (t CO <sub>2</sub> )	394	385	394
Cost Increase w/ Incentive ('000 EUR)	906,013,639	514,380,788	906,013,639
Cost Increase w/o Incentive ('000 EUR)	906,151,639	514,495,788	906,151,639
Operating Cost Increase ('000 EUR)	35,976,857	799,058	35,976,857
Feedstock Cost Increase ('000 EUR)	869,904,467	513,066,990	869,904,467
Setup Cost ('000 EUR)	270,314	629,740	270,314
Incentive ('000 EUR)	138,000	115,000	138,000

The optimal solution mix of the government using the objective function in Eq(1) and the industry using the objective function in Eq(2) are shown in Table 2. The solutions for the leader and follower individually are shown in the first two columns. The government (leader) prefers ATJ because its overall environmental impact, considering the variable emissions and setup emissions, are lower. This maximizes the overall reduction in carbon emissions that are desired by the government from the status quo (CAF setup). On the other hand, the industry (follower) prefers FT as the overall monetary expenditures are less, considering the incentive that is provided also. In other words, the increase in the overall cost that the follower needs to spend from the status quo to the FT setup is less. Both solutions serve as the optimal strategy for the government and industry individually and does not consider the interaction of both parties.

The developed auxiliary model assumes that both parties compromise to minimize the trade-offs one party has over the other. The optimal solution mix for the Approximate Stackelberg Strategy using the objective functions in both Eq(1) and Eq(2) is choosing ATJ, which is coincidentally the same solution as what the leader prefers for this illustrative case. This considers the trade-offs between both parties together with their interaction using the interactive fuzzy optimization algorithm in Eq(11) to Eq(14). The solutions can be seen in the rightmost column of Table 2. The government would prefer ATJ because of the carbon reduction it brings. Although this

would not yield the most benefit for the industry, as it would be when FT is chosen by both parties, this would be a just compromise considering the environmental objective of the government and the economic objective of the industry. The incentives that are provided by the government would aid the industry to pay for the expenditures on investing in a more sustainable technology. For the illustrative case study, the follower would compromise to the leader even though the incentive is small. This mimics the real-life scenario of the incentives given to produce SAF (Waguespack, 2023).

## 6. Conclusions

Through developing a bilevel mathematical optimization model on the decision-making for SAF production, a Stackelberg game model was developed considering the government-industry interactions maximizing carbon emission reduction as the interest of the leader and minimizing cost increase as the interest of the follower. The government provides incentives to subsidize the industry to influence the latter's decision to select one that benefits the environment. The Approximate Stackelberg Solution approach was used for this study and identified incentives from the government are an effective strategy if they want to encourage the industry to produce SAF by covering up for the costs that they need to shoulder to invest in SAF production. The optimal solution mix from the illustrative case study resulted in a reduction of 20,109 M t CO<sub>2</sub> reduction and an increase in cost of 906,014 M EUR over a period of ten years. While the model was executed with a shorter computational time, the study could be carried out more accurately if a more precise method was used. The Karush–Kuhn–Tucker conditions is a promising methodology that future studies can consider due to its short computational time and accuracy. Future studies can also consider using system dynamics to model the dynamic behaviour of government-industry interactions. Researchers can consider incentives as a decision variable over a parameter to check the optimal incentive to benefit the industry better as current incentives are not enough. It is also highly recommended to collaborate with SAF production firms to obtain accurate costs and benefits from decisions.

### Nomenclature

#### Parameters

$IC_i$ – Government Incentive, EUR	$OVE_i$ – Current Plant Variable Emissions, kg CO <sub>2</sub>
$NFC_i$ – New Setup Feedstock Cost, EUR/m <sup>3</sup>	$SAF$ – Amount of SAF to produce, m <sup>3</sup>
$NOC_i$ – New Setup Operations Cost, EUR/m <sup>3</sup>	$SC_i$ – New Setup Setup Cost, EUR
$NVE_i$ – New Setup Variable Emissions, kg CO <sub>2</sub>	$SE_i$ – New Setup Setup Emissions, kg CO <sub>2</sub>
$OFC_i$ – Old Setup Feedstock Cost, EUR/m <sup>3</sup>	
$OOC_i$ – Old Setup Operations Cost, EUR/m <sup>3</sup>	

#### Decision Variables

$x_i$ – Binary Variable for leader's selection of SAF pathway, dimensionless	$z_i$ – Binary Variable for leader's and follower's selection of SAF pathway, dimensionless
$y_i$ – Binary Variable for follower's selection of SAF pathway, dimensionless	

#### Auxiliary Model Variables

$\lambda$ – Linearizing Dummy Variable for Approximate Stackelberg Strategy	
$F$ – Follower's Stackelberg Objective Function	$L$ – Leader's Stackelberg Objective Function
$F^*$ – Follower's Worst-Case Objective Function	$L^*$ – Leader's Worst-Case Objective Function
$F^{**}$ – Follower's Best-Case Objective Function	$L^{**}$ – Leader's Best-Case Objective Function

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