

Exploring Feasible Carbon Trading Scheme via Graph-Theoretic Method

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Carbon trading is a mechanism that allows entities to sell or purchase credits which indicate the right to emit carbon dioxide. This strategy is intended to incentivize industrial reduction of carbon emissions and allow firms to outsource decarbonization if internal measures are not viable. The credits are deemed retired after it has been redeemed by a given entity. The feasible trading schemes are constrained by two parameters: (i) temporal availability of carbon credits and (ii) temporal demand of carbon credits. In this work, P-graph – a graph-theoretic framework – is employed to generate optimal trading schemes. The P-graph model is constructed in the form of a cascade model, with the aim of minimizing two critical cost aspects simultaneously: the carbon penalty incurred by the consumers due to non-compliance with emissions goal (at the above-pinch region) and profit loss experienced by the supplier stemming from the costs associated with generating credits (at the below pinch region). The effectiveness of the proposed Carbon Trading P-graph (C_Trading P-graph) is demonstrated through illustrative case studies. This model aids in informed decision-making, facilitating strategic planning in the carbon credit market.

1. Introduction

The latest IPCC report indicates that achieving the goals set in the Paris Agreement requires reaching net-zero emissions by mid-century (IPCC, 2022). The optimal strategy for achieving net-zero emissions involves a combination of methods, including transitioning to renewable energy sources, increasing energy efficiency, managing energy demand, and actively removing carbon dioxide from the atmosphere (IPCC, 2022). Carbon dioxide removal (CDR) using negative emissions technologies (NETs) can provide a way to economically reduce greenhouse gas emissions on a large scale through different physical, chemical, and biological processes (Minx et al., 2018). However, the question remains on how to finance CDR efforts. To finance these extensive decarbonization strategies, economic mechanisms like carbon taxes or carbon trading can be employed (Hashim et al., 2022). Carbon trading enables entities to buy or sell credits representing the authorization to emit carbon dioxide. This approach can be used to motivate firms to commit to carbon emissions reduction. In the case where internal measures are not viable, firms can also incentivise emissions reduction by participating in carbon trading. The trading of CDR credits is necessary but not sufficient for reaching the Paris Agreement goals (Daggash and MacDowell, 2019), and CDR markets can motivate the commercialization of these technologies (Hickey et al., 2023). Thus, the modeling of CDR trading is a research area of growing interest. Studies have reported models for CDR credit trading using optimization (Magenthirarajah et al., 2022) and game theory (Babonneau et al., 2021). Mathematical modeling has also been used to describe the impact of different cost models on corporate profitability considering CDR credits (Tsai et al., 2023). Carbon offsetting networks considering various CDR interventions have been modeled in the literature (Choi, 2023). A game theoretic optimization approach for carbon trading with blockchain has been developed (Kazi and Faruque Hasan, 2024). Pinch analysis has also been utilized to model peer-to-peer carbon trading (Migo-Sumagang et al., 2022).

In addition to graphical and mathematical programming approaches, process graph or P-graph can be used to model carbon trading. P-graph was developed to address process network synthesis (PNS) as well as PNS-like problems (Friedler et al., 2022). P-graph algorithms are rigorously derived based on five axioms common to all PNS problems (Friedler et al., 1992a). Maximal structure generation (MSG) enables the algorithmic assembly of an error-free and non-redundant superstructure known as the maximal structure (Friedler et al., 1993). Solution structure generation (SSG) allows complete enumeration of structurally feasible networks for any PNS problem (Friedler et al., 1992b). Accelerated branch-and-bound (ABB) uses implicit information to accelerate the solution of linear PNS models (Friedler et al., 2022). One advantage of P-graph over mathematical programming is its enhanced computational efficiency (Klemeš and Varbanov, 2015). Additionally, P-graph is advantageous because of its visualization and capacity to automatically generate alternative solutions, thus assisting in the decision-making process (Friedler et al., 2022). Several studies have applied P-graph in modeling carbon management networks including renewable energy networks, energy efficient systems, carbon capture and storage, and negative emission technologies. For example, P-graph has been used in modeling biomass energy supply chains (Lam et al., 2010) and integrated biorefineries (Sangalang et al., 2021). However, the P-graph framework has yet to be applied to carbon trading.

In this work, the P-graph framework is used to generate and optimize a carbon trading model. The feasibility of matching in the current model is constrained by two parameters: temporal availability of carbon credits and temporal demand for carbon credits (Migo-Sumagang et al., 2024). The model simultaneously minimizes two critical aspects: the carbon penalty incurred by the consumers and the profit loss experienced by the supplier.

2. Problem Statement

The problem statement is formally stated as: Given a set of sources $i \in I$ which indicates suppliers of carbon credits who can generate fixed credits throughout a given time interval $t \in T$. These credits can be traded with a given set of demands $j \in J$ which can be referred to as buyers of carbon credits who need extra credits to offset their emissions throughout a given time interval. With the constraint that the credits can only be debited and traded after they are created (i.e., pre-paid credit is not allowed), the proposed problem is expressed in the form of a cascade model (Figure 1). Generally, the model aims to determine the optimal trading scheme which minimizes (i) the carbon penalty due to non-compliance with emissions goals and (ii) potential profit loss attributed to the unutilized credits.

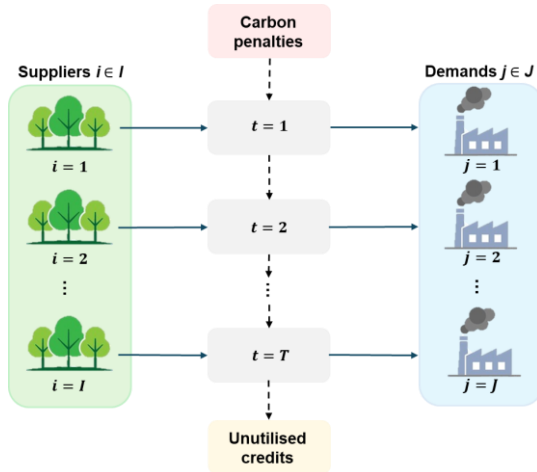


Figure 1: General superstructure of the proposed problem

3. Methodology

This work proposes to use P-graph to address the presented carbon trading problems. To ease the understanding, both mathematical formulation and P-graph representation are presented.

3.1 Mathematical Formulation

The carbon credit flow cascade can be mathematically expressed as Eq(1), where δ_t refers to the net carbon credits at each time interval t which can be cascaded into the subsequent time interval; δ_{t-1} refers to the net carbon credit cascaded from the previous time interval; while $F_{j,t}$ and $F_{i,t}$ show the carbon credit demand and carbon credit supply at each time interval t , respectively.

$$\delta_t = \delta_{t-1} + \sum_i F_{i,t} - \sum_j F_{j,t} \quad (1)$$

To ensure the carbon credit can only be traded after it is produced, all δ_t in the model must be kept positive. The resultant $\delta_{t=0}$ indicates the carbon credit deficit which potentially leads to carbon penalty; while $\delta_{t=T}$ denotes the carbon credit surplus which is not utilised (loss in profit). Therefore, the objective function of the model is set to minimise the total cost (TC) as indicated in Eq(2):

$$TC = \delta_{t=0} C^{Penalty} + \delta_{t=T} C^{Loss} \quad (2)$$

where $C^{Penalty}$ refers to the unit penalty cost for excess emissions; while C^{Loss} indicates the unit cost required to generate carbon credit.

3.2 P-graph representation

The cascade model expressed in Section 3.1 is represented as a P-graph model (Figure 2). It is constructed using P-graph Studio v5.2.5.0 (P-graph Studio, 2024).

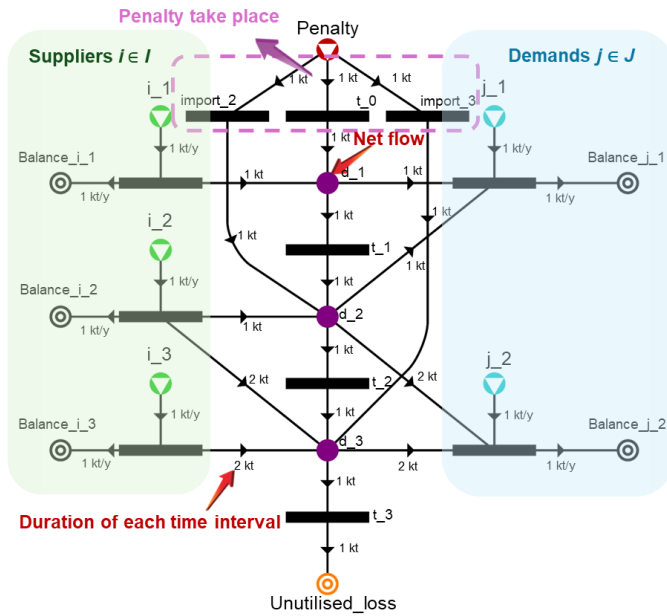


Figure 2: Illustration of $C_Trading$ P-graph model

The illustrative examples involve three-time intervals (i.e., “ t_1 ”, “ t_2 ” and “ t_3 ”, where the respective duration is inserted as the value at arcs) with three suppliers denoted as “ i_1 ”, “ i_2 ” and “ i_3 ” and two demands denoted as “ j_1 ” and “ j_2 ”. The time interval is set based on the time when (i) suppliers are generating credits and (ii) demands are consuming credits. For instance, “ i_1 ” which is assumed to be only generating credits during “ t_1 ”, is connected to “ d_1 ”. In case there is surplus credit, it can be cascaded down for usage in the subsequent time intervals. In contrast, “ i_2 ” that is connected to both “ d_2 ” and “ d_3 ” indicates that the credits are produced during “ t_2 ” and “ t_3 ”. Since it merely starts generating credit in “ t_2 ”, the credit is not allowed to back-cascade to “ t_1 ”. The annual supply of credits and the annual consumption of credits are inserted to the supplier (green-coloured) and demand (blue-coloured) nodes, respectively; while the $C^{Penalty}$ and C^{Loss} are inserted to the material node labelled as “ $Penalty$ ” and operating node placed before the material nodes labelled as “ $Unutilised_Loss$ ” nodes, respectively. With this setting, the model will tend to keep both of these nodes at minimum so that the overall economic value is maximized. It is worth noting that the additional operating nodes added after the “ $Penalty$ ” is used to indicate the time when penalty is to take place. In other words, the latest time to source external credits (in order to meet the emissions goal) can be determined by allocating different cost weight to these operating nodes (i.e., smaller costs for later intervals).

4. Case Study

Three case studies adopted from Migo-Sumagang et al. (2024) are used to illustrate the effectiveness of the proposed $C_Trading$ P-graph model.

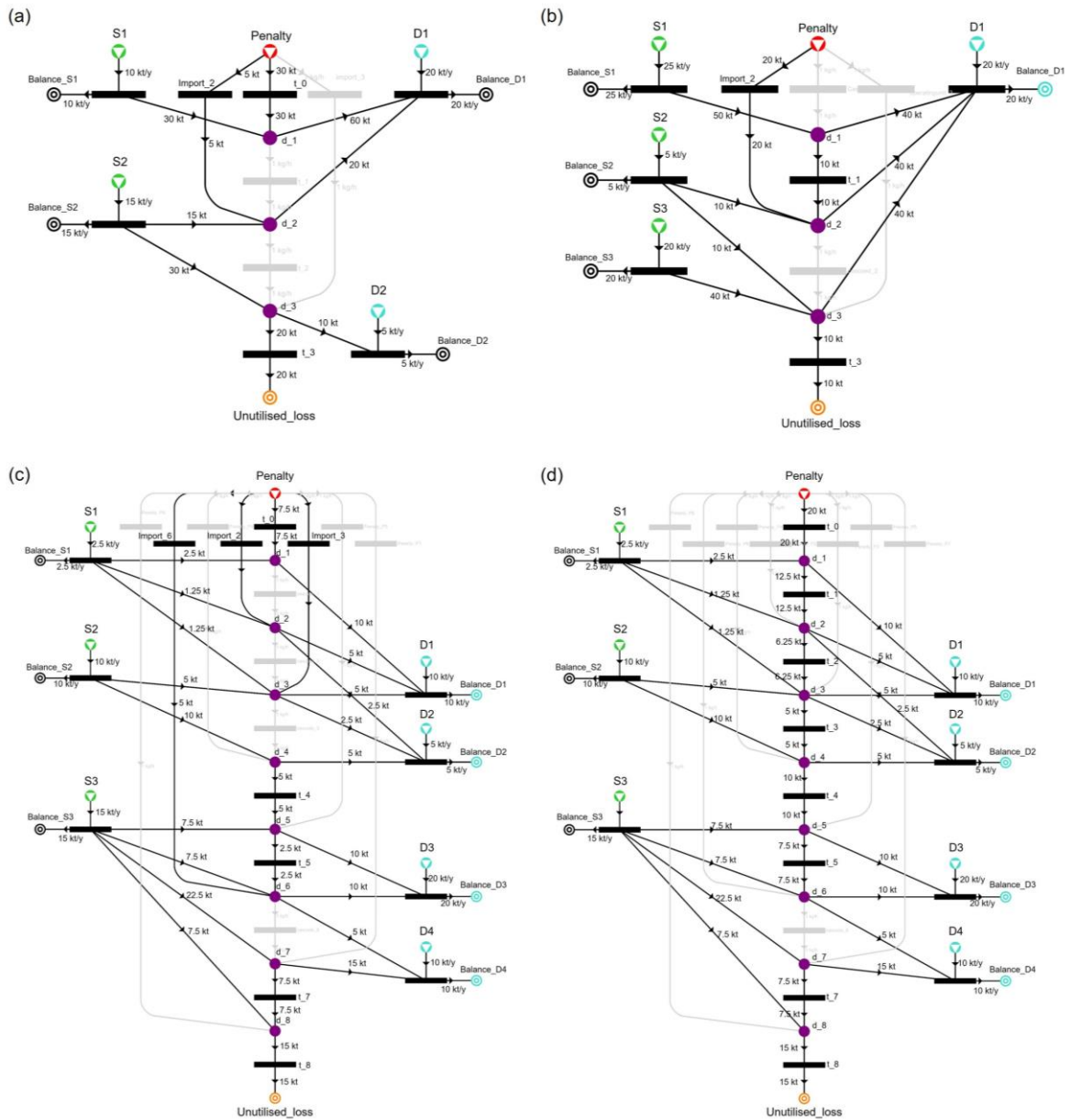


Figure 3: Optimal carbon trading network for (a) illustrative case 1, (b) illustrative case 2, and (c) illustrative case 3; (d) Alternative carbon trading network for illustrative case 3

4.1 Illustrative Case 1

A two-supplier-two-consumer problem is demonstrated in Table 1. The problem can be decomposed into three periods, i.e., “ t_1 ” (Year 0 – 3), “ t_2 ” (Year 3 – 4) and “ t_3 ” (Year 4 – 6).

Table 1: Limiting data for illustrative case 1

Supply/Demand	Start time (y)	End time (y)	Carbon credit production/ consumption (kt/y)
S1	0	3	10
S2	3	6	15
D1	0	4	20
D2	4	6	5

Figure 3(a) shows the optimised carbon trading network for this case, where the identified $\delta_{t=0}$ and $\delta_{t=T}$ are 35 kt and 20 kt. Based on the model, it can be seen that the external source of carbon credit is needed since Year 0 or else D1 would experience penalty due to insufficient credit (consumption rate of D1 is greater than

the production rate of the supplier - S1 in " t_1 "). The model also shows that the demand of D1 is jointly supplied by S1 and S2, while S2 is the sole supplier for D2.

4.2 Illustrative Case 2

This case contains 3 suppliers and 1 consumer, where the respective information can be found in Table 2. Similar to the former case, this problem can be decomposed to three time intervals, i.e., " t_1 " (Year 0 – 2), " t_2 " (Year 2 – 4) and " t_3 " (Year 4 – 6). As shown in Figure 3(b), the minimum $\delta_{t=0}$ associated for this case is 20 kt, where it must be sourced externally at the latest by the second interval, specifically at Year 2.67 (i.e., latest time = $2 + 20/(20 - 5)$); where "2" refers to the start time of second interval, "20" in the numerator refers to the credit channeled from " t_1 "; "20" and "5" in the denominator denote the carbon credit consumption (for D1) and production (from S2) rates for the involved suppliers and consumers at " t_2 ". There are 10 kt of unutilized credit ($\delta_{t=7}$) at the end of the investigated periods. It is worth noting that, based on the result, one can deduce that the surplus can possibly be contributed by S2 and/or S3, while the actual distribution is not certain.

Table 2: Limiting data for illustrative case 2

Supply/Demand	Start time (y)	End time (y)	Carbon credit production/consumption (kt/y)
S1	0	2	25
S2	2	6	5
S3	4	6	20
D2	0	6	20

4.3 Illustrative Case 3

A larger case with 3 suppliers and 4 consumers (Table 3), is illustrated in this case, where a total of eight time intervals (i.e., " t_1 " (Year 0 – 1), " t_2 " (Year 1 – 1.5), " t_3 " (Year 1.5 – 2), " t_4 " (Year 2 – 3), " t_5 " (Year 3 – 3.5), " t_6 " (Year 3.5 – 4), " t_7 " (Year 4 – 5.5), and " t_8 " (Year 5.5 – 6)) can be formed. The optimal carbon trading network is shown in Figure 3(c), where a minimum of 20 kt of external source of credit ($\delta_{t=0}$) must be sourced externally to meet the emissions target of all customers involved. Based on the result, the 15 kt must be sourced to cover deficit in the first three intervals (Year 0 – 2); while the remaining 5 kt must be sourced at the latest by " t_6 ", particularly in Year 3.67 (i.e., latest time = $3.5 + 2.5/(30 - 15)$); where "3.5" refers to the start time of " t_6 ", "2.5" in the numerator refers to the credit channeled from " t_5 "; "30" and "15" in the denominator denote the carbon credit consumption (for D3 and D4) and production rates (from S3)). In this illustrative case, 15 kt of carbon credit supplied by S3 are not redeemed. In addition to the optimal network that attempt to source the external credit as late as possible, the C_Trading P-graph model is also capable of generating other alternative solutions (e.g., alternative network shown in Figure 3(d) suggests sourcing all necessary credits at the beginning of project instead). The identification of these networks is pivotal especially when the seasonal unit price of credit of each supplier is known. With this info, the optimal sourcing time of credit can therefore be determined.

Table 3: Limiting data for illustrative case 3

Supply/Demand	Start time (y)	End time (y)	Carbon credit production/ consumption (kt/y)
S1	0	2	2.5
S2	1.5	3	10
S3	3	6	15
D1	0	2	10
D2	1	3	5
D3	3	4	20
D4	3.5	5.5	10

5. Conclusions

A graph theoretic approach for planning carbon trading has been developed in this work. The P-graph framework originally developed for PNS problems was adapted to the problem of time-constrained matching of vendors and buyers of carbon credits, based on the assumptions that the credits can only be sold after they are generated. Note that in the case where a source of carbon credits is available for a range of time periods, this work assumes those credits will be equally distributed between time periods where they are available. Multiple case studies were used to illustrate the usefulness of this approach to practical scenarios. Future work can be directed towards analysing the impact of different carbon credit distribution within their availability period on the

carbon trading scheme. Game-theoretic extensions can also be developed to account for the self-interested behaviour of individual companies.

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